No matter what score you are aiming for, knowing how to deal with exponents is crucial. Generally, questions based solely on exponents lie in the range of 600-650 and are one of the favorite topics of GMAT (but that's just my opinion). It is also a topic which is very easy once you get the 'hang of it' but generates instant dislike until you don't. I am sure many of you hate the sordid looking expressions/equations such as

$(2^a \times 4 \times 3^{-4} \times 3^b)/(3^4 \times 2^2) = 8^{-4} \times 729$

But of course, I don't blame you. Sadly, exponents and roots are basics that we should be experts in – whether we are working on Number Systems, Algebra or even Geometry! So let's roll!

Today I will just discuss the basic rules for people who are averse to exponents (I will try and explain these rules in detail to help them get comfortable). For roots and tricky questions, watch out for the next posts.

Some Basic Mathematics:

$3^4 = 3 \times 3 \times 3 \times 3$ (When a number is multiplied by itself, you raise it to a power)

$3 \times 4 = 3 + 3 + 3 + 3$ (When a number is added to itself, you multiply. In essence, multiplication is just addition. But that's a topic for another day.)

In the first case above, the '3′ is called the base and the '4′ is called the exponent.
It follows then that 3^2 would be just 3×3. So now, if we multiply 3^4 by 3^2, what do we get? $(3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3^6$. The indices just got added!

Rule 1: $a^m \times a^n = a^{(m+n)}$
When you multiply two numbers that have the same base, their exponents get added.

Example: $4^5 \times 4^3 = ?$

$4^5 \times 4^3 = 4^{(5+3)} = 4^8$

Now tell me, what happens when we divide 3^4 by 3^2?
$3^4 / 3^2 = (3 \times 3 \times 3 \times 3)/(3 \times 3) = 3 \times 3$

But we know that $3 \times 3 = 3^2$. So essentially, $3^4 / 3^2 = 3^{(4-2)} = 3^2$

Rule 2: $a^m / a^n = a^{(m-n)}$
When you divide two numbers that have the same base, the exponent of the divisor gets subtracted from the exponent of the dividend.

Example: $4^5 / 4^3 = ?$

$4^5 / 4^3 = 4^{(5-3)} = 4^2$

Let's look at another important case.

$4^4 = (2^2)^4 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^8$

The exponents just got multiplied!

Rule 3: $(a^m)^n = a^{mn}$

Example: $4^5 \times 2^8 = ?$

$4^5 \times 2^8 = (2^2)^5 \times 2^8 = 2^{10} \times 2^8 = 2^{(10+8)} = 2^{18}$

Rule 4: For any number a, $a^0 = 1$

Example: $3^0 = 1$

Now that we have discussed Multiplication and Division, we need to think about Addition and Subtraction.
What happens when we add 3^5 to 3^3? What is $3^5 + 3^3$? Can I still add the exponents? Think. (3 x 3 x 3 x 3 x 3) + (3 x 3 x 3) = 243 + 27 = 270. We cannot play with the exponents when dealing with Addition and Subtraction. But we can take a common factor. Let me show you:

Example: $3^5 + 3^3 = ?$

$3^5 + 3^3 = 3^3 \times (3^2 + 1) = 27 \times 10 = 270$
Here, we have taken three 3s out from both the numbers and added the rest. It saves us time. We don't need to calculate 3 × 3 × 3 × 3 × 3. Similarly, we can handle subtraction too.

Let's take a quick question now.

Question:

Given $(2^n \times 4 \times 2^4)/8 = 1$, what is the value of n?

Solution:

We know that 4 is 2^2 and 8 is 2^3.

We get: $(2^n \times 2^2 \times 2^4)/2^3 = 1$

Here, all the terms have the same base i.e. 2 and the terms are multiplied/divided. Therefore, the exponents can be added/subtracted.

$2^{(n+2+4)} / 2^3 = 1$

$2^{(n+2+4-3)} = 2^{(n+3)} = 1$

$2^{(n+3)} = 2^0$

Wait a minute! From where did we get 2^0? Since we know that $2^0 = 1$, if we have 1, we can write it as 2^0. A term with base 2 will be equal to 1 only if the exponent is 0.

Now, since the bases on both sides of the equation are same, the exponents should also be the same.

n + 3 = 0

n = -3

Note: We used the long method to solve this question since we wanted to discuss the application of various rules. You

can use faster approaches once you are comfortable with these basic rules.

More on negative exponents in the next post.